

# Principles of Mathematical Modeling Applied to Animal Science

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## Abstract

One of the characteristics by which we can estimate the stage of development of a certain discipline is its degree of mathematization. Thus, Galileo Galilei said that „The great book of nature can be read only by the one who knows the language in which this book was written and this language is the mathematics”. We understand by this the extent of interdisciplinary use of the mathematical ideas and techniques. It is obvious, from the viewpoint of the respective discipline, that a high degree of mathematization does not show a high intrinsic value. Situations exist in which important mathematical developments did not lead to progress in a certain discipline, such as the endowment with sophisticated equipment’s did not result implicitly in notable results. Truly important is the effective contribution and not the sophistication or elegance of the used mathematical instrument. A relatively simple mathematical idea can have an unexpected effect if used with skill. On the other hand, very elegant mathematical considerations may be of no use for the actual problems of that particular discipline.

**Keywords:** mathematical modelling, animal science, animal performance.

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One can distinguish a number of 4 stages in the contribution of mathematics to the development of a scientific discipline:

**Stage 1:** Data collection, analysis and interpretation

This is the inlet of mathematics in the scientific work. In certain situations there may be huge collections of data obtained by observation. Take for example the data collected by Gregor Mendel on plant crossing, etc.

In other situations, there is little available data, such as the data on rare natural phenomena. It is a very difficult task to decide which data are necessary and how to collect them.

On the other hand this is essential. The lack of significant data delays the general development of a scientific discipline and in particular, its mathematization. From the viewpoint of the interaction with mathematics, data collection and analysis belongs to statisticians and informaticians. Many mathematical researches

were challenged by problems raised by the processing of actual data.

**Stage 2:** Quantitative formulation on scientifically principles and empirical laws

When expressed in a thorough manner the data collections can render evident remarkable regularities. These regularities can lead to “empirical laws” of the investigated field, which, for the beginning are nothing but convenient modalities of summarizing the observation. The degree of precision of an empirical law can vary from vague to exact. Generally, in the physical sciences, the empirical laws can have very exact enunciation. In the biological or social sciences these laws are much less precise and their legitimacy can easily be attacked.

When we talk about empirical laws we must not forget that they are frequently inferred on the basis of general observations. The data being processed, the law can only be valid for an “average” situation possibly inexistent.

Rendering evident empirical laws as expressive summary of some experiments can be one of the objectives of the scientific investigator. For the theoretician scientist they are half way between the phenomenon and its explanation, and for the mathematician they can be the trigger of his interest for a particular domain.

The identification of an empirical law is followed in the process of mathematization of a particular science by the development of a structure able to explain this law. Such a structure together with a list of correspondences between the mathematical symbols and the objects of the concrete situation is what we call a mathematical model. (The term "model" as derived from the Latin word *modus* = measure, involves a change of scale in the representation of the concrete situation. From this basal meaning we note today only that we are searching for a different scale of thinking or understanding, therefore of abstraction).

**Stage 3:** Regarding the mathematical development of a model we can perceive two trends concerning the very operation of modelling. A first trend is the exaggerated simplification of the concrete situation which has the advantage of increasing possibilities for effective mathematical developments, allowing the use of the model in a wide range of different contexts.

A second trend, more and more present, due to the development of the computing capacity, is to include in the model as many as possible characteristics of the concrete situation and to use numeric methods to obtain the results.

The value of a model, namely whether the model is adequate or not to reality is usually a very complex task. The model will contain a number of parameters, concrete quantities which will have to be expressed in order to compare the predictions with the empirical data. Many models, otherwise acceptable in the first beginning, were abandoned because they were based on parameters that could not be experimented.

The model, as a logical structure, rises most of times problems interesting from the mathematical viewpoint, which can lack significance for the concrete situation that led to the model. The development of these problems can lead to the appearance of large chapters of mathematics.

One of the big advantages of mathematical modelling is that structures or other very similar ones can appear in completely different scientific

contexts, and a problem which can completely lack significance in a context can be of particular importance for another. Thus, the development of a model without immediate applicability is not a worthless effort and we must not blame those mathematicians who develop a so-called unavailable section of applied mathematics.

The value of a model in relation to its contribution to the study of the modelled concrete situation is determined by its degree of adequacy, namely the manner in which the predictions of the model agree with the observations. The construction of a model is a dialectical process. A first variant of the model which proves to be inadequate is changed, and the new predictions are confronted with the reality. This refining of the model can take place several times before achieving a convenient adequacy.

The role of the mathematician in this stage is merely to develop the model mathematically.

**Stage 4:** Utilization of the mathematical models for the progress of the scientific knowledge

A mathematical model once developed and its predictions inferred by reasoning, we can ask ourselves whether we can use this material past its simple adequacy. In many cases the model and its predictions can lead us either to discovering and rendering evident unknown aspects or to clarifying others partially known.

In the previous stage, the existing data were used to validate the model and now the model tells us where to look in order to obtain new data. Modern physics offers many examples which illustrate this situation. In exchange, in the biological sciences, the models which lead to the progress of knowledge are extremely rare. An example is Mendel's laws used in genetics which allowed a thorough access in the mechanisms of heredity both for the individual and for the populations. In fact most of the researchers involved in biological sciences are either in Stage 1 or 2. There are controversies regarding the extent to which mathematics will be some day an as efficient language for other sciences as it was for physics.

It is not excluded that the answer to that question is negative and it will thus be necessary to create new mathematical structures able to develop the instruments required by the modelling of the biological science. But irrespective of the answer it is a fact that today life sciences begin to play for

mathematics the role of problem generator, role fulfilled so far by the physical sciences.

### Basic terminology

The mathematical models defined as sets of mathematical relations which describe the behaviour of a certain system can be classified in a simple manner, a mathematical model being either static or dynamic, determinist or stochastic, empirical or Newtonian.

A static model has no time variable while a dynamic model has a time variable (explicitly or implicitly). Mathematics recognizes a natural representation of the dynamic systems as differential equations.

A determinist model provides defined predictions for various quantities without any probability distribution associated to it. A stochastic model, on the other hand, contains allegory elements or distributions of probability anticipating not only the expected value of a quantity, but also its variation. The appearance of a stochastic model reflects often the lack of knowledge of reality and may provide a false vision on reality. The stochastic models are also very difficult to handle technically and can become very complex in a short time.

The empirical model mainly describes, while the Newtonian model tries to offer a description accompanied by the understanding of its causality and studies the mechanisms present in a given system (namely, the manner in which the parts composing the system interact, like in a machine). It is developed by studying the system's structure, by dividing it in the composing parts and by trying to analyze the behaviour of the whole system from the viewpoint of the individual components and of the interactions between them. The approach is therefore the analytical reduction.

An empirical model is based on observation and experiment and not indispensably on a preconceived biological theory. The empirical approach consists in examining the experimental data and in establishing the equation or system of equations that can be used as a mathematical model; they will thereafter be suited to the given data. The approach concerns therefore the use of models too in order to describe, explaining the inherent variation between them, without dealing especially with the given mechanism.

The empirical approach is not compulsory reduction. The modelling specialist of Newtonian type will wish to elaborate his models before the experiment, thinking to the possible mechanisms and inferring their consequences with the assistance of a model; the experiment will test the hypothesis, and perhaps favouring one mechanism at the expense of another.

On the other hand, the empirical approach will infer the mechanism after the experiment takes place and the data are analyzed. Empirism is in fact the traditional and dominant approach in the agricultural sciences, on the basis of which the conclusions regarding the mechanism are drawn after analysis of the experimental evidence. The Newtonian and empirical approaches are both valid forms of scientific research, although the empirical models usually have a limited ability of explaining. In agriculture they must be used in parallel such as they are used in most of the other scientific fields, because one does not replace the other. In solving a problem the Newtonian approach is generally used by mathematicians and mathematician physicists, and the empirical approach is used by the statisticians.

It is a fact that more and more domains of biology use mathematical models and that the range of used mathematical models is increasing continuously.

Nevertheless, there are aspects of the interaction between mathematics and biology where no progresses were made; these aspects are revealed only when the initial stages of development are concerned, not the final product.

It is therefore well to have in mind the following scheme when the success or failure of any attempt of cooperation between biology and mathematics is evaluated:

1. The biological competence of the mathematician (including his ability to listen and to discuss with his biologist colleague, and ability to assimilate the problems of biology that he hears or reads about);
  - a) for a subject on "fundamental" biology he will influence the biological content or the resulting theory;
  - b) for a subject on "applied" biology he will influence the practical use of the used theoretical methods,
2. The mathematical competence of the biologist (including his ability to present the biological issues to his mathematician colleague and his

ability to moderate his mathematical fantasy, as well as capacity to listen, read and interpret the mathematical results);

- a) for any subject on "fundamental" biology he will determine the adequate character or its lack for the mathematical method used as a reflection of the biological understanding;
- b) for any subject on "applied" biology he will help in choosing the best fitted mathematical models (such as avoiding the formula of the linear regression when the regression is clearly a curve).

In connection to the above there are certain aspects such as:

3. Selection of the research objective.
4. Strategy of approaching the subject after it was selected.
5. Main target of the research, level of certitude, amount of details developed in the proposed response to the proposed problem.

Our concern refers to some typical aspects of the interaction between mathematics and biology and not to the internal problems of each field.

This is why we did not mention explicitly some obvious requirements such as the mathematical competence of the mathematician or the biological competence of the biologist.

Because our interest is primarily connected to the performance of the biomathematical methods as participants in the biological discovery, it is useful to remember the possible meanings of the term "biological discovery".

Today biology is clearly oriented, and observations prove that, towards theory. Some biologists would go as far as to assert that were there are no experiments and observations, we have no biology. This line of thinking leads to denying the status of "contributor to the biological discovery" for any activity not originating from new experiments and observations.

Likewise, any activity originating in such experiments or at least if these experiments are accepted as "significant" by a significant number of biologists they would receive the status of "contributor to the biological discovery". In the same manner, similarly to any activity leading to new ways of conducting experiments, these ways consist either in new instruments to carry experiments or in new methods to design and analyze, with the condition that these instruments or methods be considered as improvements as compared to the previous ones.

According to these criteria, Darwin's theory of evolution (which is of course mathematics in itself) was clearly a major contribution to the biological discovery; decades of fruitful work in compared anatomy and population genetics followed, to name only two of the domains involved in Darwin's theory.

Similarly, the assembly of statistical "designing and inferring methods should be considered as a major contribution to the biological discovery; R.A. Fisher did not develop only a large number of constant interactions with the agricultural purposes, but also included many research projects in biologic sciences which include applications of statistics.

The statistical methods have doubtless a mathematical origin.

In a wider sense, almost every experiment that is preceded in a specific manner by thinking is thus a contributor to the biological discovery as indeed expected or hoped. Any time this process of thinking comprises mathematical elements (as might happen each time thinking is done by a person competent in mathematics) mathematics brings its contribution. We use above the words "new" and "significant". An experiment is "new" when it was not done before and is considered as "significant" when enough reputed persons cheer it.

The concept of "new" despite the apparent simplicity of its definition is completely vague; on one hand, when the experiment conducted previously at 35°C is now conducted at 35.5°C, it will generally be regarded as not being new (except for the situation in which the result is unexpectedly), on the other hand among the experiments recognized as being new, there are a lot having many characteristics in common with previous experiments.

The next concept "significant" is even more perfidious. Consider for example the following story: in 1874 before obtaining his doctorate J.H. van Hoof published an 11 pages paper (in Dutch) containing the theory of the asymmetrical carbon atom, opening thus the way for understanding the stereoisomers and their applied compounds. In the following year this paper appeared in French (1875) and in 1876 it was translated in German because an important man, J. Wislicenus (teacher of chemistry) considered it to be of "epochal importance". In spite of all these another pre-eminent person, H. Kolbe (teacher of chemistry in

Leipzig), who had just become famous for his paper in the field of synthesis (the first one who synthesized the acetic acid from its elements), criticized the paper virulently.

Kobe being older and more famous proves that the contributions which historically proved to have extremely important signification, can have initially huge difficulties in being recognized as such.

Another example: in 1859 the president of the Linean Society of London, T. Homson Bell reviewing the papers presented before the society in the preceding year noted regretfully that he did not find any historical discoveries among its scientific departments. Nonetheless, on July 1, 1859 Ch. Darwin and A. Wallace read before the society their paper "On the tendencies of the species to form varieties"

In the same way, Mendel's paper on genetics (1865) was virtually not noted for about half century; there was no reputed person who, as expert in genetics to assert that the paper is significant, because Mendel was the only existing geneticist at that time.

Many authors of dissertations not noted so far, go ahead without the Nobel prize, while many papers buried in old publication can very well remain there.

Thus, when we must judge if the mathematical methods have contributed to the biological discoveries, we must be aware, among other, of the possibility that a major contribution is considered as significant, but it will be indifferent to the future generation.

In the first case a mathematical method will not be recognized properly; in the second case the undue recognition will have to be withdrawn in the future.

The subsequent biological experiments resulted in the analysis of the concept of competitions and to the discovery that rather small changes in the environmental conditions (such as temperature) can result in the dominant species losing its dominance, and to many other discoveries: the importance of the genetic differences, of the separation of the physical stance in the years of different spreading of the non homogenous mediums etc. and all this due to two short equations !!

Which are the causes due to which the mathematical modelling and simulation are

relatively so little developed in the agricultural sciences?

1. This activity is often limited because it is complex (we understand by this the difficulties existing in the apparition and survey of an interdisciplinary team too).

2. The required information tends to be incomplete. The specialist often uses recommendation taken from books (the insufficient documentation on the most adequate mathematical methods to be used).

The computer is the major element for solving these problems, the complexity not being an obstacle anymore, and the information gaps from the systems can be identified immediately and if it is the case, can be filled with the equivalent existing at the particular moment until appearance of new, improved investigations.

3. The nutritionists, physiologists, biochemists refuse generally to be involved. This is why the efforts of many scientists doing modelling lack the biological credibility.

4. The quality of the mathematical knowledge is often poor. A large deal of the mathematic facts used is supplied by persons lacking a formal training in the mathematical sciences. In consequence the developed methods are often hard to use, formulated in a confusing manner and sometimes inconsistent mathematically.

The publications of nutrition science often do not approach the real problems. The purely empirical descriptions are not sufficient any more; it is now necessary to dwell on the knowledge of the biological mechanisms. It is naive to study nitrogen metabolism in rumen independently of carbon metabolism; same, the energy and protein inputs are arbitrary limited when animal performance is determined. The research was thus directed to the possible contribution of the mathematical modelling, of the kind used so successfully in the physical sciences.

If we want the agricultural modelling to progress we must consider it as an activity of cooperation between physiologists, nutritionists, biochemists, etc. and mathematicians, the required mathematical experience having thus to be put at the disposition of the biologists.

#### **Possible contributions**

Mathematical modelling can contribute to the agricultural scientific knowledge in many ways, some of which are presented below.

1. Biological hypothesis expressed in mathematical language can offer a description on quantitative understanding of the biologic problems.
2. The stimulation of the mathematical modelling can prove to be a conceptual frame which can contribute to the discovery of unknown domains, and can stimulate the appearance of new ideas and of experimental approaches on more rigorous basis. The scientific researcher can waste the resources solving false problems, and the specialist in development can transmit this information to the producers, who might even use them.
3. A mathematical model can provide a way through which the data accumulated by research can be put to use by the farmers in a readily accessible form.
4. The practical advantages of the methods produced by search can be studied with a model, stimulating thus the use of improved methods.
5. The mathematical modelling determines the appearance of experiments which are less ad-hoc, because with the help of them it is easier to design experiments which to answer certain research requirements, or to dissociate between alternative mechanisms.
6. Within a system with more components, the model offers a manner of gathering the knowledge on the component parts in order to give them a correct image on the behaviour of the whole system. Thus, the information regarding the energy content of a concentrate is useless in the absence of the data on the prices of the end products.
7. Modelling can contribute to securing a strategic and tactical backing for a research program, justifying the activity of the scientists and promoting collaboration.
8. A model can be an efficient means to summarize data, and a prudent method of interpolation and extrapolation.
9. The data becoming more and more precise, but more and more expensive, a model can use data more efficiently, sometimes using them better.
10. The power of prediction of an efficient model can be used in many ways: research, development, managing, planning priorities. For example, a model can offer answers to questions such as: "What if ...?"; Which would be the consequences of cutting the concentrate intake in cattle on the milk yield and forage requirement? Which would

be the effect of increasing the weaning weight of calves on the subsequent meat production? etc.

In connection to the concrete activity in biomathematics we note there main directions: in genetics, physiology and biotechnology.

The differences existing between the three domains can also be found in the mathematical models mostly used. In genetics and breeding the population studies require probabilistic methods while in the other two domains the determinist mathematical disciplines seem to be more adequate.

As concerns the problems connected to the interdisciplinary team we must say that any attempts of interdisciplinary study are useful. For the mathematicians the presence of another specialist for dialogue is essential. The quality of the model depends on the quality of the dialogue, and the quality of the dialogue depends on the quality of men and on the working atmosphere.

## References

- [1.] Burlacu, Gh., Burlacu, R., Cavache, A., Potentialul productiv al nutreturilor si folosirea lor in hrana animalelor de ferma, Editura CERES, Bucuresti, 2002, I.S.B.N. 973-40-0541-3
- [2.] Burlacu, R., Biostatistica, Editura Ceres, Bucuresti, 2007, I.S.B.N.: 978-973-40-0758-5
- [3.] Burlacu, R., Mathematical models of applied biology of some metabolic processes, Progress in research on energy and protein metabolism, EAAP publication, Wageningen Academic Publishers, 2003, p. 139-146
- [4.] Burlacu, R., Cavache, A., Surdu, I., Funcții de creștere aplicate în științele vieții, Editura Cartea Universitară, Bucuresti, 2004, I.S.B.N.:973-7813-00-6
- [5.] Dent, J.B., Blackie, M.J., Systems simulation in agriculture, Applied Science Publishers LTD, London, 1974
- [6.] Doucet, P., Sloep, P.B., Mathematical modeling in the Life Science, Ed. Ellis Horwood, New York London Toronto Sydney Tokyo Singapore., 1992
- [7.] Emmans, G.C., Animal growth and feed intake, A collection of papers 1981-1990, SAC, Edinburgh., 1992
- [8.] France, J., Thornley, J.H.M., Mathematical models in agriculture, Butterworths, London Boston Durban Singapore Sydney Toronto Wellington, 1986